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A Data-Driven H-Infinity Controller Design with Non-Common Lyapunov Matrices for the Active Structural Control Having Saturated Actuators

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***Abstract*—This paper presents a data-driven H-infinity controller for active vibration control in structural systems having saturated actuators. The data-driven approach addresses parameter uncertainties by eliminating the need for system identification. The full-block S-procedure is used to formulate a convex optimization problem in the form of linear matrix inequalities (LMIs), though additional constraints may introduce conservatism. To mitigate this, the dilation technique with non-common Lyapunov matrices is employed, reducing conservatism and achieving a 12.7% lower H-infinity norm compared to common Lyapunov matrices. A seismically excited three-storey structure is used to validate the method. Simulations based on real-time data from the Kobe earthquake show that the proposed synthesis effectively reduces vibrations while control inputs never become saturated.**

***Index Terms*—data-driven control, active structural control, non-common Lyapunov matrix, linear matrix inequalities, saturated actuators**

I. INTRODUCTION

An accurate mathematical model is essential for model-based control methods. However, modeling errors in complex systems often lead to parameter uncertainties, reducing control performance and potentially causing instability under seismic excitation. Researchers have proposed various robust control methods for active structural systems [8], [17]. Robust optimal control studies employ uncertainty modeling techniques such as linear fractional transformation (LFT) and norm-bounded representations. El Heraiki et al. [7] used LFT-based dynamic output-feedback control to handle structural uncertainties and design a robust $\mathcal{H}_\infty$ controller for a two-storey building with an active mass damper. Xu et al. [13] addressed norm-bounded uncertainties in a nine-storey building, achieving disturbance attenuation and robust stability through a discrete-time LMI-based optimal controller. While these model-based approaches prioritize robustness and stability, they often introduce conservatism in control design.

Model-free control methods can be an alternative to solving parameter uncertainty problems in active structural systems. Khalatbarisoltani et al. [9] proposed a method for active structural control by combining reinforcement learning algorithms and gain-scheduling fuzzy controllers, as demonstrated in an experimental study using past earthquake records. Dworakowski and Mendrok [4] developed a reinforcement learning–based artificial neural network to reduce the earthquake-induced vibrations in an unknown model. Despite significant results, these methods involve a two-step procedure of system identification and controller design.

Data-driven control leverages data collected from the system to design controllers without requiring explicit modeling. Particularly in complex systems that are difficult to model mathematically, this one-step procedure enhances the applicability of the control method. Recent advances in data-driven control theory built on the work of Willems et al. [16], who demonstrated that input and noise signals collected from the system could cover all trajectories when the signals are persistently exciting. De Persis and Tesi [3] developed a method to stabilize linear quadratic regulator and feedback systems by parameterizing linear feedback systems, and showed that such problems can be formulated and solved using LMIs. Later, Berberich et al. [1] designed a new robust controller using finite-length input and state trajectories, assuming that the upper bound of the disturbance input is known. This technique ensures closed-loop stability and satisfies performance constraints in linear time-invariant (LTI) systems.

Structural systems are complex, making mathematical modeling challenging. Data-driven control eliminates the need for system matrices, offering a valuable approach for active structural control. To our knowledge, only a few model-free applications for structural systems exist in the literature. Therefore, this paper proposes a data-driven $\mathcal{H}_\infty$ controller having saturated control inputs using the full-block S-procedure. Unlike existing methods, the proposed technique employs non-common Lyapunov matrices, reducing common variables and yielding less conservative solutions. Simulations on a three-storey structure using Kobe earthquake data show improved $\mathcal{H}_\infty$-norm reduction and effective disturbance attenuation while control inputs never become saturated.

The rest of the paper is structured as follows: Section 2 presents the problem formulation. Section 3 describes our data-driven $\mathcal{H}_\infty$ controller. Section 4 includes the simulation studies on three-storey structural systems. Finally, in Section 5, concludes the paper with discussions.

Notations throughout the paper are standard. Lowercase italic characters represent vectors or scalars, while uppercase letters represent matrices. An $m \times m$ dimensional identity

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matrix is denoted by I_m. $\mathbb{R}^{i\times j}$ describes the set of real numbers of size $i\times j$. The transpose of matrix G is represented by G^T. $\mathsf{He}\{P\} = P + P^T$ defines the Hermitian matrix for all $P \in \mathbb{R}$. Symmetric blocks in matrices are denoted by $*$. While $\mathsf{diag}\{\cdot\}$ indicates block diagonal matrices, and $\mathsf{conv}\{\mathcal{S}\}$ describes the convex hull of the finite set $\mathcal{S}$. The trace of the matrix P is defined by $\mathsf{trace}\{P\}$. A matrix inequality $A \succeq (\preceq)\, 0$ defines that A is a symmetric positive (negative) semi-definite. $x_i \le y_i$ holds for each $i = 1,\dots,n$ of x, y. Finally, the ellipsoid associated with matrix P is defined as $\mathcal{E}(P,r) \triangleq x \in \mathbb{R}^n : x^T P x \le r$, under the condition that $P = P^T \succeq 0$.

II. Problem Formulation

Consider the following LTI system described by the plant dynamics:

$$\mathcal{T}\left\{\quad \left[\begin{array}{c} x_{k+1} \\ \hline z_k \end{array}\right] = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C & D_w & D_u \end{array}\right] \left[\begin{array}{c} x_k \\ \hline w_k \\ u_k \end{array}\right], \qquad x_0 = 0 \right. \tag{1}$$

where $x \in \mathbb{R}^n$, $w \in \mathbb{R}^{m_w}$, $u \in \mathbb{R}^{m_u}$ and $z \in \mathbb{R}^c$ are the state vector, disturbance inputs, control inputs and controlled outputs, respectively. For simplicity, x^+ symbolizes x_{k+1} while x represents the state signal at time instant k. We define an N-length sequence of state vectors $\{x_k\}_{k=0}^{N-1}$ with $X = \begin{bmatrix} x_0 & x_1 & \cdots & x_{N-1}\end{bmatrix}$ and its time-shifted (delayed) version with $X_+ = \begin{bmatrix} x_1 & x_2 & \cdots & x_N\end{bmatrix}$. Similarly, the disturbance and control input sequences are $W = \begin{bmatrix} w_0 & w_1 & \cdots & w_{N-1}\end{bmatrix}$ and $U = \begin{bmatrix} u_0 & u_1 & \cdots & u_{N-1}\end{bmatrix}$. The entire state vector is considered measurable in real time, and the states, disturbances, and control inputs are bounded as follows:

$$\mathcal{X} = \{x : Tx \le \mathbf{1}\}, \tag{2}$$
$$\mathcal{W} = \left\{w \in \mathbb{R}^{m_w} : w_k^T w_k \le \bar{w}^2,\ \forall k \ge 0\right\}, \tag{3}$$
$$\mathcal{U} = \{u : Hu \le \mathbf{1}\}, \tag{4}$$

where $T \in \mathbb{R}^{n_T\times n}$ and $H \in \mathbb{R}^{n_H\times n}$. In (2) and (4), $\mathbf{1}$ describes a vector whose entries are equal to 1. Note that we assume A and B_u matrices are unknown and all others are known or user-defined. Furthermore, w includes all disturbances and noises acting the system, and is assumed to satisfy the bound $\bar{w}$.

Our primary objective is to design an identification-free, data-driven controller based on a state-feedback control law which minimizes the closed-loop $\|\mathcal{T}\|_\infty$ gain of the structural system, while control and state trajectories are not saturated. Here, $\|\mathcal{T}\|_\infty$ can be defined as $\sup\limits_{\|w\|\neq 0} \frac{\|z\|_2}{\|w\|_2} \le \gamma_\infty$. Hence, the closed-loop system can be defined by applying $u_k = Kx_k$ state-feedback control law to (1):

$$\mathcal{T}_{cl}\left\{\quad \left[\begin{array}{c} x^+ \\ \hline z \end{array}\right] = \left[\begin{array}{c|c} \mathcal{A}_{cl} & \mathcal{B}_{cl} \\ \hline \mathcal{C}_{cl} & \mathcal{D}_{cl} \end{array}\right] \left[\begin{array}{c} x \\ \hline w \end{array}\right], \qquad x_0 = 0, \right. \tag{5}$$

where $\mathcal{A}_{cl} = A + B_u K$, $\mathcal{B}_{cl} = B_w$, $\mathcal{C}_{cl} = C + D_u K$ and $\mathcal{D}_{cl} = D_w$.

In order to reach a control synthesis method that does not rely on knowledge of the A and B_u matrices, we employ an initialization process over a period of N samples, where $n + m_u < N$. In this process, a generated disturbance signal sequence $W \in \mathscr{W}$ and random, rich input signal sequence U affect the system. Note that the exact values of $W \in \mathbb{R}^{m_w\times N}$ are unknown and chosen from a bounded region $\|w\|_2 \le \bar{w}$ randomly for each column. The measured states along 0 to $N-1$ generate state signal sequences X. After the $(N-1)$th sample, input-state data are used on the control synthesis method to solve a convex optimization problem. This procedure enables the design of a controller that is tailored to the system's specific characteristics and operating conditions. However, the input-state data sequences should be appropriate for describing the dynamic behaviors. The following definition outlines the criteria for appropriate data sequences.

Definition 1: Consider a discrete-time LTI $x_{k+1} = Ax_k + B_u u_k$ system. If $\left[\begin{smallmatrix} X \\ U\end{smallmatrix}\right]$ has full row rank, then the input-state data collected from this system is called persistently exciting. Persistently exciting data is essential for accurately capturing system behavior and enabling effective control design.

Lemma 1: [16] Consider a controllable LTI system given by $x_{k+1} = Ax_k + B_w w_k + B_u u_k$. $\left[\begin{smallmatrix} X \\ U\end{smallmatrix}\right]$ is always persistently exciting if the augmented data matrix $\left[\begin{smallmatrix} W \\ U\end{smallmatrix}\right]$ has full row rank. Now we are ready to define all terms that contain A and B_u with the input-state data, $\{x_k, u_k, w_k\}_{k=0}^{N-1}$. The following lemma defines the closed-loop system by using this data instead of A and B_u. Thus, the controller can obtain the control law even without any knowledge of these matrices.

Lemma 2: [1] If there exist matrices $G \in \mathbb{R}^{N\times n}$ and $K \in \mathbb{R}^{m_u\times n}$ that satisfy

$$\begin{bmatrix} I_n \\ K \end{bmatrix} = \begin{bmatrix} X \\ U \end{bmatrix} G, \tag{6}$$

then

$$A + B_u K = (X_+ - B_w W)G. \tag{7}$$

Here, $W \in \mathscr{W}$ is an unknown and bounded disturbance signal matrix. Furthermore, if $N \ge n + m$ and $\left[\begin{smallmatrix} X \\ U\end{smallmatrix}\right]$ is always persistently exciting, then there is a G that satisfies (6) for any K.

III. Data-Driven $\mathcal{H}_\infty$ Control

According to Lemma 2, we need to find a solution with bounded W to find a data-driven controller. The following lemma will help us define bounding conditions on closed-loop energy based on LMIs.

Lemma 3: [10] For a given $\alpha \in (0,1)$, the system (5) is asymptotically stable under disturbance signals satisfying $w_k^T w_k \le \bar{w}^2$ for all $k > 0$, if there exist matrices Y, L and $Q = Q^T$ such that

$$\begin{bmatrix} (1-\alpha)Q & * & * \\ \alpha\mathcal{A}_{cl}Y & \begin{pmatrix} \alpha(Y + Y^T) \\ -Q \end{pmatrix} & * \\ 0 & \alpha\mathcal{B}_{cl}^T & \alpha I \end{bmatrix} \succ 0, \tag{8}$$

$$\begin{bmatrix} Q & * \\ \mathbf{e}_i^T TY & \left(\frac{1}{\bar{w}}\right)^2 \end{bmatrix} \succ 0, \quad \forall i = 1,\dots,n_T. \tag{9}$$

$$\begin{bmatrix} Q & * \\ \mathbf{e}_i^T HKY & \left(\frac{1}{\bar{w}}\right)^2 \end{bmatrix} \succ 0, \qquad \forall\, i = 1, \ldots, n_H. \quad (10)$$

Here, $\mathbf{e}_i$ denotes an n-dimensional column vector with a 1 in the i-th entry, while all other entries are zero. Moreover, all constraints in (2), (3) and (4) are satisfied.

Remark 1: In Lemma 3, (8) is derived from the condition $\Delta V + \alpha V - \alpha w^T w \prec 0$, where V is a Lyapunov function and $\Delta V = V_{k+1} - V_k$. This condition further leads to $\Delta V \prec \alpha \left(w^T w - V\right) \prec \alpha \left(\bar{w}^2 - V\right)$ under the constraint (3). As observed, this inequality defines a controlled and invariant ellipsoid spanned by $\bar{w}$. Any trajectory starting within the ellipsoid cannot leave it until ΔV reaches zero or higher values. Therefore, system trajectories should never become saturated to ensure stability. Hence, satisfying both (8) and the ellipsoidal bound (9) guarantees system stability. Additionally, finding a solution under the constraint (3) is feasible by taking W as an uncertainty, bounded using the full-block S-procedure.

To design compatible with Lemma 3, an $\mathcal{H}_\infty$-norm constraint should be described in a dilated form, which can be given as the following bounded real lemma.

Lemma 4: [2] The system (5) has an $\mathcal{H}_\infty$-norm from w to z less than $\gamma_\infty > 0$ ($\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$), if there exists matrices Y and $Q = Q^T$ such that

$$\begin{bmatrix} Q & * & * & * \\ Y^T \mathcal{A}_{cl}^T & \left(Y + Y^T - Q \right) & * & * \\ \mathcal{B}_{cl}^T & 0 & I & * \\ 0 & \mathcal{C}_{cl} Y & \mathcal{D}_{cl} & \gamma_\infty^2 I \end{bmatrix} \succ 0. \quad (11)$$

In order to obtain a solution with the unknown but bounded disturbance data matrix, $W \in \mathscr{W}$ will be taken as an uncertainty by using the full block S-procedure approach [14]. This technique helps us to obtain a less conservative data-driven solution that can be applied with Lemma 5. At first, we should describe a multiplier matrix Φ that satisfies,

$$\begin{bmatrix} W^T \\ I \end{bmatrix}^T \underbrace{\begin{bmatrix} \Psi & \chi \\ \chi^T & \Pi \end{bmatrix}}_{\Phi} \begin{bmatrix} W^T \\ I \end{bmatrix} = W \Psi W^T + \mathsf{He}\{W\chi\} + \Pi \prec 0, \qquad \forall\, W \in \mathscr{W}. \quad (12)$$

Furthermore, W and unstructured multiplier matrix $\mathcal{G}$ can be defined in the LFT form as

$$W \star \underbrace{\left[\begin{array}{c|c} \mathcal{G}_{11} & \mathcal{G}_{12} \\ \hline \mathcal{G}_{21} & \mathcal{G}_{22} \end{array}\right]}_{\mathcal{G}} \triangleq \mathcal{G}_{22} + \mathcal{G}_{21} W (I - \mathcal{G}_{11} W)^{-1} \mathcal{G}_{12}. \quad (13)$$

Here, $W \star \mathcal{G}$ is called well-posed, if $(I - \mathcal{G}_{11} W)$ is invertible for all $W \in \mathscr{W}$. Finally, the following is our main lemma that is used for obtaining the data-driven synthesis.

Lemma 5: [6] $W \star \mathcal{G}$ is well-posed and

$$\mathsf{He}\{W \star \mathcal{G}\} \triangleq W \star \mathcal{G} + (W \star \mathcal{G})^T \succ 0, \ W \in \mathscr{W}, \quad (14)$$

holds if, and only if,

$$\begin{bmatrix} \mathcal{G}_{21} \Pi \mathcal{G}_{21}^T + \mathsf{He}\{\mathcal{G}_{22}\} & \mathcal{G}_{21} \Pi \mathcal{G}_{11}^T + \mathcal{G}_{21} \chi^T + \mathcal{G}_{12}^T \\ * & \Psi + \mathcal{G}_{11} \Pi \mathcal{G}_{11}^T + \mathsf{He}\{\mathcal{G}_{11} \chi^T\} \end{bmatrix} \succ 0, \quad (15)$$

and there exist a block-structured matrix Φ that satisfies (12).

As a result, our main result obtained by data-driven closed-loop system identification using Lemma 3 and Lemma 4 is as follows:

Theorem 1: For a given $\alpha \in (0, 1)$, the closed-loop system (5) has $\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$ and satisfies (2) and (4) subjected to all disturbance signals satisfying (3), if there exists matrices M, Y, Π_1, Π_2, Ψ_1, Ψ_2, χ_1, χ_2 and $Q_j = Q_j^T$ of appropriate dimensions such that $XM = Y$, and

$$\begin{bmatrix} W^T \\ I \end{bmatrix}^T \begin{bmatrix} \Psi_i & \chi_i \\ \chi_i^T & \Pi_i \end{bmatrix} \begin{bmatrix} W^T \\ I \end{bmatrix} \prec 0, \quad i = 1, 2, \quad (16)$$

$$\begin{bmatrix} (1-\alpha) Q_j & * & * & * \\ \alpha X_+ M & \begin{pmatrix} \alpha (Y + Y^T) - Q_j \\ + B_w \Pi_1 B_w^T \end{pmatrix} & * & * \\ 0 & \alpha B_w^T & \alpha I & * \\ \alpha M & -\chi_1 B_w^T & 0 & \Psi_1 \end{bmatrix} \succ 0, \quad j = 1, \quad (17)$$

$$\begin{bmatrix} Q_j & * \\ \mathbf{e}_i^T TY & \left(\frac{1}{\bar{w}}\right)^2 \end{bmatrix} \succ 0, \quad \forall i = 1, \ldots, n_T, \quad j = 1, \quad (18)$$

$$\begin{bmatrix} Q_j & * \\ \mathbf{e}_i^T HUM & \left(\frac{1}{\bar{w}}\right)^2 \end{bmatrix} \succ 0, \qquad \forall\, i = 1, \ldots, n_H, \quad j = 1, \quad (19)$$

$$\begin{bmatrix} Q_j + B_w \Pi_2 B_w^T & * & * & * & * \\ M^T X_+^T & Y + Y^T - Q_j & * & * & * \\ B_w^T & 0 & I & * & * \\ 0 & CY + D_u UM & D_w & \gamma_\infty^2 I & * \\ -\chi_2 B_w^T & M & 0 & 0 & \Psi_2 \end{bmatrix} \succ 0, \quad j = 2. \quad (20)$$

Then, the controller can be constructed as $u = UMY^{-1}x$.

Proof 1: Consider (8) with the definition of $Q \triangleq Q_j$ for $j = 1$. Application of Lemma 2 to the closed-loop system (5) using $A_{cl} = (X_+ - B_w W) G$ and $\mathcal{B}_{cl} = B_w$, and defining $M \triangleq GY$ gives

$$\begin{bmatrix} (1-\alpha) Q_j & * & * \\ \alpha \left\{ (X_+ - B_w W) M \right\} & \begin{pmatrix} \alpha (Y + Y^T) \\ -Q_j \end{pmatrix} & * \\ 0 & \alpha B_w^T & \alpha I \end{bmatrix} \succ 0, \quad (21)$$

which can be rewritten in the LFT framework as

$$\underbrace{\begin{bmatrix} (1-\alpha) Q_j & * & * \\ \alpha X_+ M & \alpha (Y + Y^T) - Q_j & * \\ 0 & \alpha B_w^T & \alpha I \end{bmatrix}}_{\mathcal{G}_{22} + \mathcal{G}_{22}^T} + \mathsf{He} \left\{ \underbrace{\begin{bmatrix} 0 \\ -B_w \\ 0 \end{bmatrix}}_{\mathcal{G}_{21}} W (I - \underbrace{0}_{\mathcal{G}_{11}} W)^{-1} \underbrace{\begin{bmatrix} \alpha M & 0 & 0 \end{bmatrix}}_{\mathcal{G}_{12}} \right\} \succ 0. \quad (22)$$

This matrix inequality is in the same form as (14). Therefore, the application of (15) on (22) for $i = 1$ gives (17).

In addition to the same closed-loop and M definitions, the consideration of (11) by the definition of $Q \triangleq Q_j$ for $j = 2$ gives

$$\begin{bmatrix} Q_j & * & * & * \\ M^T(X_+ - B_w W)^T & \left(Y + Y^T - Q_j\right) & * & * \\ B_w^T & 0 & I & * \\ 0 & CY + D_u UM & D_w & \gamma_\infty^2 I \end{bmatrix} \succ 0, \quad (23)$$

with $K = UG$ that comes from Lemma 2. Finally, (23) can be represented in LFT form as

$$\underbrace{\begin{bmatrix} Q_j & * & * & * \\ M^T X_+^T & \left(\ Y + Y^T - Q_j\ \right) & * & * \\ B_w^T & 0 & I & * \\ 0 & CY + D_u UM & D_w & \gamma_\infty^2 I \end{bmatrix}}_{\mathcal{G}_{22} + \mathcal{G}_{22}^T}$$

$$+ \text{He}\left\{ \underbrace{\begin{bmatrix} -B_w \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathcal{G}_{21}} W(I - \underbrace{0}_{\mathcal{G}_{11}} W)^{-1} \underbrace{\begin{bmatrix} 0 & M & 0 & 0 \end{bmatrix}}_{\mathcal{G}_{12}} \right\} \succ 0. \quad (24)$$

Similarly, the application of (15) on (24) for $i = 2$ yields (20). Moreover, the definition of Lemma 5 leads to an additional constraint (16). For $I = XG$, the equivalent condition $XM = Y$ should satisfy. For $j = 1$, considering the definitions of K and M to (10) gives (19). Finally, there is no change in the bounding condition on the system states for $j = 1$. This completes the proof. ∎

Remark 2: As seen in Theorem 1, (21) relies on the exact W values. To find a generalized solution for all $W \in \mathscr{W}$, we will use the convex-hull relaxation technique defined in [10].

Remark 3: Finding a common solution for both input saturation and minimizing $\mathcal{H}_\infty$-norm problems can cause a conservatism problem. The dilated LMIs allow us to use non-common Lyapunov matrix Q_j in the control method. Thus, the number of common variables is reduced in different control problems and the solution set of LMIs is expanded. This situation could help us to find a less conservative solution.

IV. Simulation Studies

This section demonstrates the application of the proposed data-driven controller to a three-storey structural system under seismic excitation. SeDuMi solver [15] and Yalmip parser [12] are used to solve the LMIs given in Theorem 1, and all simulations are carried out with MATLAB.

Aiming to reduce conservatism with the utilization of non-common Lyapunov matrix, two different cases are considered in the simulation studies of the data-driven controller. First, the Lyapunov matrix is taken equal as $Q_1 = Q_2$ (Case 1) in Theorem 1. Then, Theorem 1 is performed for $Q_1 \neq Q_2$ (Case 2). Additionally, a continuous-time model-based $\mathcal{H}_\infty$ controller (MBC), derived from Corollary 4 in [11] and formulated using dilated LMIs, is employed to further evaluate the performance of the proposed data-driven controller.

In the simulation studies, we consider the three-storey structural system shown in Figure 1 [5]. Even though the data-driven controller does not require A and B_u system matrices, these are needed for the simulation of the system. Here, y_0, y_1,

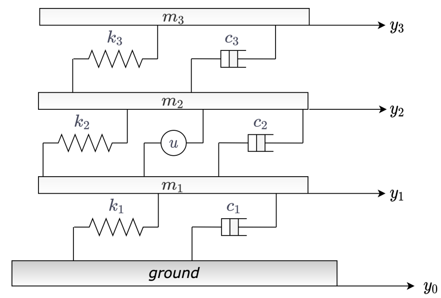

Fig. 1. Three-Storey Structural System

y_2 and y_3 are displacements of each storey. When $i = 1, 2, 3$ represents the storeys, m_i, c_i and k_i are mass, damping and stiffness coefficients, respectively. The motion of structural system can be defined by the following differential equations

$$\underbrace{\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix}}_{\ddot{y}} + \underbrace{\begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_2 & c_3 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix}}_{\dot{y}}$$

$$+ \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_2 & k_3 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{y} = F_u u(t) + F_w w(t), \quad (25)$$

where M, C, $K \in \mathbb{R}^{n_0}$ are the mass, damping and stifness matrices. While $F_u = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}^T \in \mathbb{R}^{m_u \times n_0}$ shows the position of the controller, the weights of the disturbance input is defined by $F_w = \begin{bmatrix} -(c_1\dot{y}_0 + k_1 y_0) & 0 & 0 \end{bmatrix}^T \in \mathbb{R}^{m_w \times n_0}$. Finally, the state-space representation of the system is given as follows:

$$\underbrace{\frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} 0_{n_0 \times n_0} & I_{n_0} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}}_{\bar{A}} \underbrace{\begin{bmatrix} y \\ \dot{y} \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0_{n_0 \times m_w} \\ -M^{-1}F_w \end{bmatrix}}_{\bar{B}_w} w(t)$$

$$+ \underbrace{\begin{bmatrix} 0_{n_0 \times m_u} \\ -M^{-1}F_u \end{bmatrix}}_{\bar{B}_u} u(t) \quad (26)$$

The discrete-time version of the system in the form of (1) can be obtained by sampling the continuous-time system with a sampler having a sampling period of $T_s = 0.02$ [s].

For simulation studies, the system parameters are taken as $m_1 = 56$, $m_2 = 34$, $m_3 = 23$ [ton]; $c_1 = 29.818$, $c_2 =$

17.89, $c_3 = 11.928$ [kNs/m]; $k_1 = 1602$, $k_2 = 960$, $k_3 = 640$ [kN/m] [5]. To achieve effective disturbance attenuation performance, controlled output matrices are chosen as $C = I_n$, $D_w = 0_{n\times m_w}$ and $D_u = 0_{n\times m_u}$.

We assume that the bounds on the states, disturbance and control input are selected as $x_{max} = \begin{bmatrix}0.04 & 0.04 & 0.04 & 0.4 & 0.4 & 0.4\end{bmatrix}^T$, $u_{max} = 20$ [kN] and $w_{max} = \begin{bmatrix}0.28 & 0.28\end{bmatrix}^T$ respectively, in the controller design. Note that the disturbance bound should cover all possible disturbances to be controlled. The parameter α is selected as $\alpha = 0.5$ through a line search to find the minimum value of γ_∞. The number of samples is chosen as $N = 25$ for a successful learning performance in the data-driven controller design. Using the initial values, the application of Theorem 1 gives γ_∞ and K values of the data-driven controller as shown in Table I. Additionally, MBC results can be found in Table I. Note that the exact values of the $\mathcal{H}_\infty$-norm, denoted as $\gamma_{\infty t}$, are computed by applying the data-driven controller gain to the exact system model. When the system matrices are unknown, the learning performance depends on how close $\mathcal{H}_\infty$-norm obtained by the data-driven synthesis method is to the exact $\mathcal{H}_\infty$-norm. For Case 1 and Case 2, the exact values of $\mathcal{H}_\infty$-norm are obtained as $\gamma_{\infty t} = 27.5$ and $\gamma_{\infty t} = 24$, respectively.

TABLE I
γ_∞ AND CONTROLLER GAINS

Controller	γ_∞	Controller Gains (K)
Data-Driven ($Q_1 = Q_2$)	27.47	$10^5 \times \begin{bmatrix}-1.2806 & 1.3728 & -0.1922 & -0.0160 & -1.8226 & -1.0393\end{bmatrix}$
Data-Driven ($Q_1 \neq Q_2$)	24.01	$10^5 \times \begin{bmatrix}-4.1348 & 3.6689 & -0.3289 & -0.0634 & -3.8709 & -2.4780\end{bmatrix}$
MBC	18.31	$10^5 \times \begin{bmatrix}2.7427 & -4.8987 & 8.9953 & 2.5133 & -2.0064 & -2.3112\end{bmatrix}$

As shown in the numerical results, 12.7% lower $\mathcal{H}_\infty$-norm can be obtained by employing a non-common Lyapunov function compared to its common version. Thus, closer γ_∞ values can be obtained with Case 2 compared to the MBC. In comparison with MBC results, 50% higher $\mathcal{H}_\infty$-norm is obtained with Case 1 when it is reduced to 31% with Case 2. These outcomes show that the non-common selection of the Lyapunov matrix allows us to reduce conservatism in the control problem.

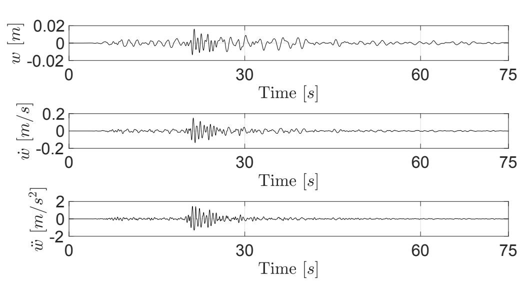

Fig. 2. Kobe Earthquake in Japan (1995)

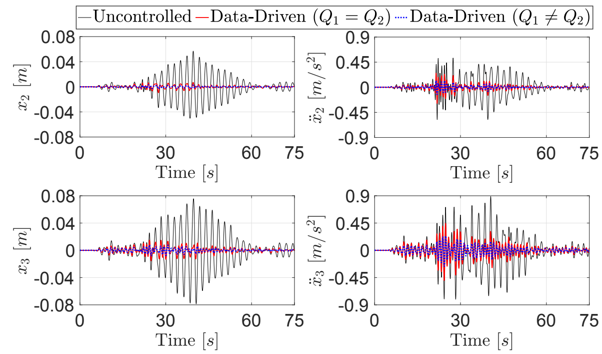

Fig. 3. Trajectories of the Second and Third Storey of the Structural System for the Data-Driven Controller

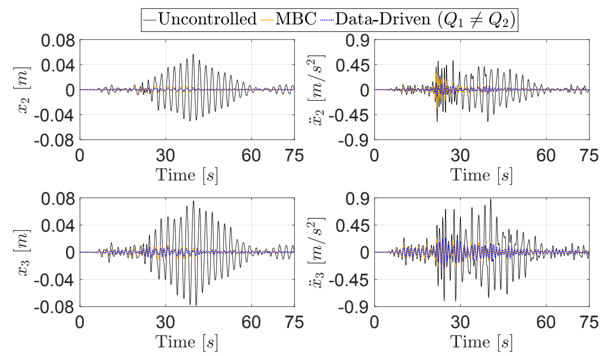

Fig. 4. Trajectories of the Second and Third Storey of the Structural System for Data-Driven Controller ($Q_1 \neq Q_2$) and MBC

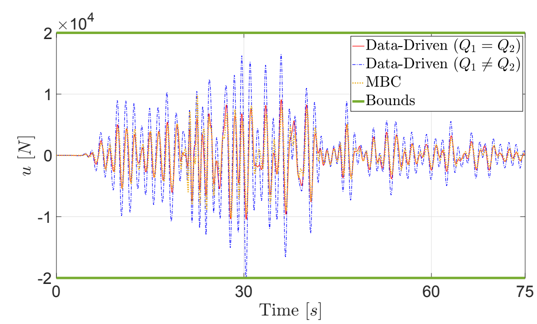

Fig. 5. Control Forces

In the simulations, the ground motion from the 1995 Kobe Earthquake is used as a disturbance input, shown in Figure 2. Under the seismic excitation, the second and third storey trajectories realized by applying the obtained K values to the system are given in Figure 3 for both data-driven controlled and uncontrolled cases. In Figure 4, the same trajectory responses of data-driven controller ($Q_1 \neq Q_2$) and continuous-time MBC are shown. Finally, Figure 5 demonstrates the control forces.

Simulation results show that the data-driven controller can reduce vibration amplitudes more successfully when $Q_1 \neq Q_2$. Moreover, the proposed technique provides disturbance attenuation performance comparable to that of the MBC, but with higher control forces. Finally, the control forces of the data-driven controller never become saturated.

The root mean square (RMS) values of the given trajectories in Figure 3 and Figure 4 are provided in Table II for numerical comparison. These numerical results support the simulations. The notable observation is that, the proposed technique can achieve slightly lower RMS values in some dynamics while causing higher control forces.

TABLE II
RMS VALUES OF THE SECOND AND THIRD STOREY TRAJECTORIES

RMS Values	x_2 [m]	$\dot{x}_2$ [m/s]	$\ddot{x}_2$ [m/s^2]
Uncontrolled	0.0167	0.0485	0.1588
Data-Driven ($Q_1 = Q_2$)	0.0020	0.0082	0.0463
Data-Driven ($Q_1 \neq Q_2$)	0.0008	0.0037	0.0222
MBC	0.0017	0.0070	0.0500
RMS Values	x_3 [m]	$\dot{x}_3$ [m/s]	$\ddot{x}_3$ [m/s^2]
Uncontrolled	0.0238	0.0713	0.2410
Data-Driven ($Q_1 = Q_2$)	0.0042	0.0198	0.1063
Data-Driven ($Q_1 \neq Q_2$)	0.0022	0.0109	0.0590
MBC	0.0031	0.0135	0.0685

V. CONCLUSIONS

This study proposes a data-driven $\mathcal{H}_\infty$ controller for active structural systems having saturated control inputs, requiring only user-defined disturbance input and performance output matrices, eliminating the need for system matrices. By employing the full-block S-procedure, the method addresses parameter uncertainty without system information but may introduce conservatism. To reduce this, non-common Lyapunov matrices are used in solving dilated LMIs, expanding the solution set. Simulations on a three-storey structure excited by the Kobe earthquake demonstrate reduced conservatism, effective vibration attenuation, and unsaturated control inputs. Future work may extend these findings to mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control.

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